# EVALUATING MPPT CONVERTER TOPOLOGIES USING A MATLAB PV MODEL

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**Abstract**

An accurate PV module electrical model is presented based on the Shockley diode equation. The simple model has a photo-current current source, a single diode junction and a series resistance, and includes temperature dependences. The method of parameter extraction and model evaluation in Matlab is demonstrated for a typical 60W solar panel.

This model is used to investigate the variation of maximum power point with temperature and insolation levels. A comparison of buck versus boost maximum power point tracker (MPPT) topologies is made, and compared with a direct connection to a constant voltage (battery) load. The boost converter is shown to have a slight advantage over the buck, since it can always track the maximum power point.

## 1 PHOTOVOLTAIC MODULES

Solar cells consist of a p-n junction fabricated in a thin wafer or layer of semiconductor. In the dark, the I-V output characteristic of a solar cell has an exponential characteristic similar to that of a diode.

When exposed to light, photons with energy greater than the bandgap energy of the semiconductor are absorbed and create an electron-hole pair. These carriers are swept apart under the influence of the internal electric fields of the p-n junction and create a current proportional to the incident radiation. When the cell is short circuited, this current flows in the external circuit; when open circuited, this current is shunted internally by the intrinsic p-n junction diode. The characteristics of this diode therefore sets the open circuit voltage characteristics of the cell.

### 1.1 Modelling the Solar Cell

Thus the simplest equivalent circuit of a solar cell is a current source in parallel with a diode. The output of the current source is directly proportional to the light falling on the cell. The diode determines the I-V characteristics of the cell.

Increasing sophistication, accuracy and complexity can be introduced to the model by adding in turn

- Temperature dependence of the diode saturation current $I_0$.

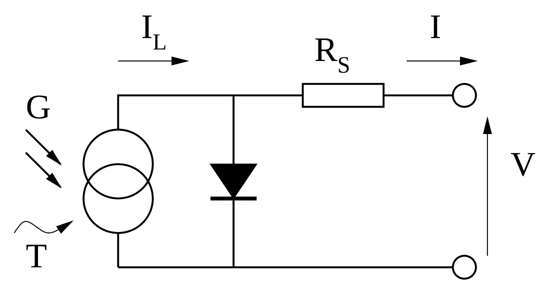


Figure 1: The circuit diagram of the PV model.

- Temperature dependence of the photo current $I_L$.
- Series resistance $R_S$, which gives a more accurate shape between the maximum power point and the open circuit voltage.
- Shunt resistance $R_P$ in parallel with the diode.
- Either allowing the diode quality factor $n$ to become a variable parameter (instead of being fixed at either 1 or 2) or introducing two parallel diodes (one with $A = 1$, one with $A = 2$) with independently set saturation currents.

For this research work, a model of moderate complexity was used. The model included temperature dependence of the photo-current $I_L$ and the saturation current of the diode $I_0$. A series resistance $R_S$ was included, but not a shunt resistance. A single shunt diode was used with the diode quality factor set to achieve the best curve match. This model is a simplified version of the two diode model presented by Gow and Manning [1]. The circuit diagram for the solar cell is shown in Figure 1.

The equations which describe the I-V characteristics of the cell are

$$I = I_L - I_0(e^{q(V+IR_S)/nkT} - 1) \quad (1)$$

$$I_L = I_{L(T_1)}(1 + K_0(T - T_1)) \quad (2)$$

$$I_{L(T_1)} = G * I_{SC(T_1,nom)}/G_{(nom)} \quad (3)$$

$$K_0 = (I_{SC(T_2)} - I_{SC(T_1)})/(T_2 - T_1) \quad (4)$$

$$I_0 = I_{0(T_1)} * (T/T_1)^{3/n} * e^{-qV_g/nk*(1/T-1/T_1)} \quad (5)$$

$$I_{0(T_1)} = I_{SC(T_1)}/(e^{qV_{OC(T_1)}/nkT_1} - 1) \quad (6)$$

$$R_S = -dV/dI_{V_{OC}} - 1/X_V \quad (7)$$

$$X_V = I_{0(T_1)} * q/nkT_1 * e^{qV_{OC(T_1)}/nkT_1} \quad (8)$$

All of the constants in the above equations can be determined by examining the manufacturers ratings of the PV array, and then the published or measured I-V curves of the array. As a typical example, the Solarex MSX60 60W array will be used to illustrate and verify the model.

The photo-current $I_L$ (A) is directly proportional to irradiance $G$ ($Wm^{-2}$). When the cell is short circuited, negligible current flows in the diode. Hence the proportionality constant in equation 3 is set so the rated short circuit current $I_{SC}$ at is delivered under rated irradiation (usually 1 Sun = $1000Wm^{-2}$). For the MSX60, $I_{SC} = 3.8A$ at 1 Sun at $T_1 = 25$°C (298K), so $I_{L(T_1)} = 3.8A/Sun$.

The relationship between the photo-current and temperature is linear (eqn. 2) and is deduced by noting the change of photo-current with the change of temperature (eqn. 4). For the MSX60, $I_L$ changes from 3.80 to 3.92A (3%) as T changes from 25 to 75°C.

When the cell is not illuminated, the relationship between the cell's terminal voltage and current is given by the Shockley equation. When the cell is open circuited and illuminated, the photo-current flows entirely in the diode. The I-V curve is offset from the origin by the the photo generated current $I_L$ (eqn 1).

The value of the saturation current $I_0$ at 25°C is calculated using the open circuit voltage and short circuit current at this temperature (eqn 6).

An estimate must be made of the unknown “ideality factor” $n$. Green [3] states that it takes a value between 1 and 2, being near one at high currents, rising towards two at low currents. A value of 1.3 is suggested as typical in normal operation, and may be used initially,

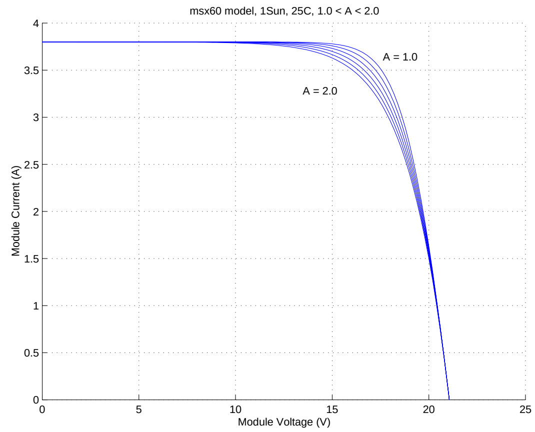


Figure 2: The Matlab model VI curves for various diode quality factors.

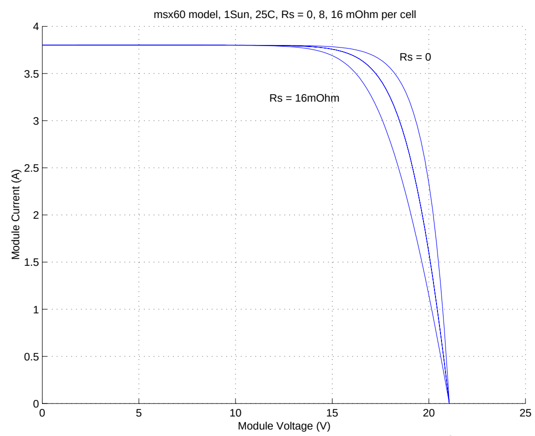


Figure 3: The Matlab model VI curves for various model series Resistances.

until a more accurate value is estimated later through curve fitting. The effect of varying the ideality factor can be seen in the MSX60 model, figure 2 – higher values soften the knee of the curve.

The relationship of $I_0$ to temperature is complex, but fortunately contains no variables requiring evaluation (eqn 5) [1].

The series resistance of the panel has a large impact on the slope of the I-V curve at $V = V_{OC}$, as seen in figure 3. Equations 7 and 8 are found by differentiating equation 1, evaluating at $V = V_{OC}$, and rearranging in terms of $R_S$ [1]. Using the values obtained from the MSX60 manufactures' curves, a value of total panel series resistance $R_S = 8m\Omega$ was calculated.

### 1.2 Matlab model of the PV module

The Solarex MSX60, a typical 60W PV module, was chosen for modelling. The module has 36 series connected polycrystalline cells. The key specifications are shown in table 1.

The model was evaluated using Matlab. The model pa-

| At Temperature | $T$ | 25 | °C |
|---|---|---|---|
| Open Cct Voltage | $V_{OC}$ | 21.0 | V |
| Short Cct Current | $I_{SC}$ | 3.74 | A |
| Voltage, max power | $V_m$ | 17.1 | V |
| Current, max power | $I_m$ | 3.5 | A |
| Maximum Power | $P_m$ | 59.9 | W |

Table 1: The key specifications of the Solarex MSX60 PV panel.

rameters are evaluated during execution using the equations listed in the previous section using the above data points contained in the script. The current I is then evaluated using these parameters, and the variables Voltage, Irradiation, and Temperature. If one of the input variables is a vector, the output variable (current) is also a vector.

The inclusion of a series resistance in the model makes the solution for current a recurrent equation (refer to eqn. 1). A simple iterative technique initially tried only converged for positive currents. The Newton Raphson method used converges much more rapidly, and for both positive and negative currents.

A listing of the Matlab script which implements the equations shown is given in Figure 4.

### 1.3 Results of Matlab PV module model

The output of the Matlab function is shown first for various irradiation levels (Fig. 5), and then for various temperatures (Fig. 6). A number of discrete data points are shown on the curves in figure 6. These are points taken directly from the manufacturer's published curves, and show excellent correspondence to the model.

## 2 COMPARING MPPT CONVERTER TOPOLOGY PERFORMANCE

### 2.1 PV systems modelled

A maximum power point tracker (MPPT) is a power electronic DC-DC converter inserted between the PV module and its load to achieve optimum matching. By using an intelligent algorithm, it ensures the PV module always operates at its maximum power point as the temperature, insolation and load vary. A number of tracking algorithms have been proven and used and a number of DC-DC converter topologies are possible. The Matlab model was developed so various peak power point tracker configurations and strategies could be compared using real irradiation data.

All these simulations were performed using the previously calculated Solarex MSX60 60W PV module model. The load was presumed to be a 12V lead acid battery, whose voltage varied from 12V (flat) to 13.8V (fully charged "float" voltage). This simple

```
function Ia = msx60i(Va,Suns,TaC)
% msx60.m   model for the MSX-60 solar array
% current given voltage, illumination and temperature
% Ia = msx60(Va,G,T) = array voltage
% Ia,Va = array current,voltage
%   G = num of Suns (1 Sun = 1000 W/m^2)
%   T = Temp in Deg C

k = 1.38e-23;  % Boltzman's const
q = 1.60e-19;  % charge on an electron

% enter the following constants here, and the model will be
% calculated based on these. for 1000W/m^2
A = 1.2;   % "diode quality" factor, =2 for crystaline, <2 for amorphous
Vg = 1.12; % band gap voltage, 1.12eV for xtal Si, ~1.75 for amorphous Si.
Ns = 36;       % number of series connected cells (diodes)

T1 = 273 + 25;
Voc_T1 = 21.06 /Ns;  % open cct voltage per cell at temperature T1
Isc_T1 = 3.80;       % short cct current per cell at temp T1

T2 = 273 + 75;
Voc_T2 = 17.05 /Ns;  % open cct voltage per cell at temperature T2
Isc_T2 = 3.92;       % short cct current per cell at temp T2

TaK = 273 + TaC;     % array working temp
TrK = 273 + 25;      % reference temp

% when Va = 0, light generated current Iph_T1 = array short cct current
%    constant "a" can be determined from Isc vs T

Iph_T1 = Isc_T1 * Suns;
a = (Isc_T2 - Isc_T1)/Isc_T1 * 1/(T2 - T1);
Iph   = Iph_T1 * (1 + a*(TaK - T1));

Vt_T1 =  k * T1 / q; % = A * kT/q
Ir_T1 = Isc_T1 / (exp(Voc_T1/(A*Vt_T1))-1);
Ir_T2 = Isc_T2 / (exp(Voc_T2/(A*Vt_T1))-1);

b =  Vg * q/(A*k);
Ir = Ir_T1 * (TaK/T1).^(3/A) .* exp(-b.*(1./TaK - 1/T1));

X2v = Ir_T1/(A*Vt_T1) * exp(Voc_T1/(A*Vt_T1));
dVdI_Voc = - 1.15/Ns / 2;       % dV/dI at Voc per cell --
                                %    from manufacturers graph
Rs = - dVdI_Voc - 1/X2v;        % series resistance per cell

% Ia = 0:0.01:Iph;
Vt_Ta =  A * 1.38e-23 * TaK / 1.60e-19; % = A * kT/q

%  Ia1 = Iph - Ir.*( exp((Vc+Ia.*Rs)./Vt_Ta) -1);
% solve for Ia: f(Ia) = Iph - Ia - Ir.*( exp((Vc+Ia.*Rs)./Vt_Ta) -1) = 0;
%  Newton's method:   Ia2 = Ia1 - f(Ia1)/f'(Ia1)

Vc = Va/Ns;
Ia = zeros(size(Vc));
% Iav = Ia;
for j=1:5;
  Ia = Ia - ...
    (Iph - Ia - Ir.*( exp((Vc+Ia.*Rs)./Vt_Ta) -1))...
    ./ (-1 - (Ir.*( exp((Vc+Ia.*Rs)./Vt_Ta) -1)).*Rs./Vt_Ta);
% Iav = [Iav;Ia];    % to observe convergence for debugging.
end
```

Figure 4: The Matlab script file used to generate the simulation results shown in this document.

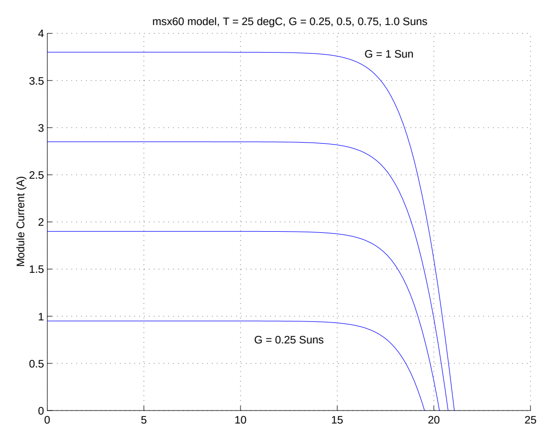


Figure 5: The Matlab model VI curves for various irradiation levels.

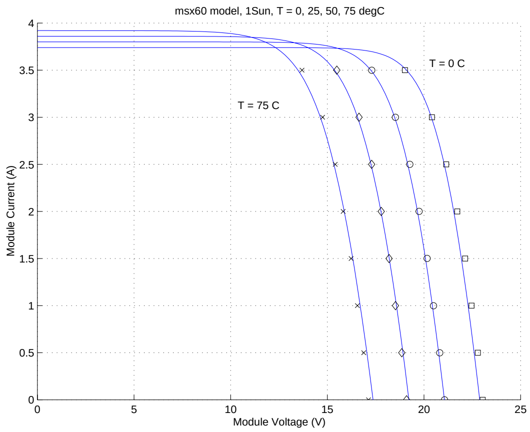


Figure 6: The Matlab model VI curves for various temperatures. The discrete data points shown are taken from the manufacturer's curves, and show excellent correspondence with the model.

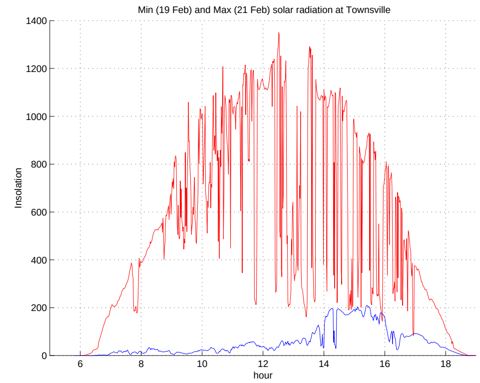


Figure 7: Instantaneous global solar radiation at Townsville for the days with a cumulative maximum (21st) and minimum (19th) radiation in February 1991.

arrangement is typical of low power stand-alone PV-battery systems powering electric fence energisers or data telemetry systems.

Three possible connections between source and load were considered.

The most typical arrangement in a simple system as described is a single series blocking diode between the PV module and the battery. A shunt regulator may be placed in parallel with the battery to prevent overcharging when the battery reaches its fully charged "float" voltage by shunting excess charge around the battery. A more typical modern regulator arrangement places a series MOSFET switch between the PV module and battery. This regulator disconnects the module when the module current falls to zero and reverses, or when the battery is fully charged. This method has the advantage of very low forward voltage drop when the MOSFET switch is closed.

These arrangements are modelled as a direct connection between module and battery. The voltage of the PV module is set by the battery voltage, and the battery current and hence power delivered is determined by the PV module operating point.

The other arrangement is the use of a MPPT DC-DC converter inserted between the PV module and battery. The ratio of the input and output voltages is controlled by varying the on-off duty cycle of the converter switching device, typically a MOSFET. Since the output voltage of the converter is fixed by the battery, the input voltage and hence module operating point is controlled by the duty cycle.

The two possible converter topologies considered are the buck converter and the boost converter.

The input voltage of a buck converter is always greater than or equal to its output voltage, so the panel voltage must exceed the battery voltage for power to flow. The maximum power point (MPP) of a nominal 12V commercial PV module is above 13V for most combinations of insolation and temperature. A buck converter can operate at the MPP under most, but not all, conditions.

The input voltage of a boost converter must lie between zero and the output voltage, for complete control of the power flow. Hence the module maximum output voltage, the open circuit voltage, must be less than the minimum battery voltage. This is possible because many modern PV modules have substrings wired out to permit bypass diodes to be added. For example the MSX60 has two series connected 18 cell strings, with a maximum open circuit voltage of approximately 11.5V each. A boost converter will always be able to operate at the panel's MPP.

A buck converter with a MOSFET switch still requires an additional diode or MOSFET to block reverse current flow when the panel voltage drops below the battery voltage. As an additional advantage, a boost converter naturally has this diode (or MOSFET) as part of its structure, which removes an additional source of voltage drop and power loss.

### 2.2 Insolation data used

The insolation data used was Global solar radiation data kindly supplied by Dr Harry Suehrcke of James Cook University [5]. The radiation measurements used here is beam and diffuse horizontal surface radiation gathered with a photovoltaic pyranometer located at James Cook University, Townsville. The data points were taken every minute for a whole year. The curves used in these simulations were for the best day (21st) and the worst day (19th) in February, based on cumulative insolation (Fig. 7).

Usually insolation data is presumed to follow a daily sinusoidal curve from dawn to dusk, with the peak value set to the value calculated for that day of the year [2]. This averaged data removes the rapid peaks and troughs caused by passing clouds. It was presumed that this introduced some potential sources of error in modelling the operation of a PV system.

Firstly at very low levels of insolation, for example during heavy cloud cover, the power flow from PV module to battery may be reduced or stop completely because the MPP or Open Circuit (OC) voltage of the module may fall below that of the battery if a simple blocking diode is used. A buck converter would be similarly affected, however, a boost converter should always be able to extract the maximum power available from the panel, even if it is low.

Secondly, rapid changes in insolation would require rapid changes of MPPT converter operating point. A converter with a slow response may spend a considerable fraction of time away from the optimum MPP. This paper will not focus on this second issue.

### 2.3 Comparison of topologies

The comparison of different interfacing options between PV module and load was done without confusing the issue by including converter losses. The DCDC converters were assumed to be 100% efficient, which many designs can approach quite closely.

Instead, matching efficiency was evaluated, which was calculated as the fraction of actual power extracted from the solar panel divided by the maximum power available at the MPP. With an appropriate setup, the MPP PV panel voltage will always be less than the battery voltage. Thus a MPPT based on the boost converter topology should always be able to operate at the maximum power point, so its matching efficiency is 100%.

The buck converter MPPT can also achieve perfect matching, so long as its input voltage always exceeds its output voltage. This may not occur for simultaneous low insolation levels and high panel temperatures. Inspection of figure 8 shows the maximum power point falls logarithmically with falling insolation, reaching 13.8V at around 150Wm$^2$ (0.15 Suns) and 50°C.

However the curve is quite flat, and at 50Wm$^2$ (0.05 Suns), the constant power curve of 2W remains in approximate tangential contact from 12V to 14V. At these very low insolation levels where power matching may not be perfect, the power available is very low anyway. The power forfeited by failing to track the MPP at these low levels overall proves to be insignificant.

The battery current and accumulated energy measured in Amp-hours for the different configurations and days are shown in Figures 9 & 11. The PV panel temperature was assumed to be a constant 50°C in both cases, which

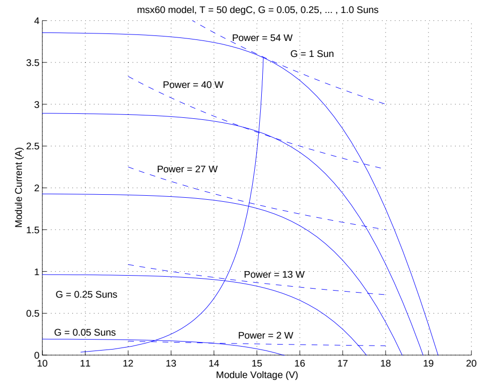


Figure 8: The Matlab model curves for various irradiance levels ($G = 50, 250, 500, 750, 1000Wm^{-2}$) at the panel's nominal operating temperature of $50°$C. The constant power curves which are tangential to each of the VI curves are shown, as well as the locus of the maximum power point voltage.

is realistic on the sunny day, but pessimistic on the dull day. Over 25 Ah is collected on the brightest day, but less than 2.5 Ah on the dullest.

On the brightest day, the boost and buck MPPT converters performed identically, collecting approximately 4% more energy than the direct connection. On the dull day, the buck connection achieves the same outcome as the direction connection, since the MPP voltage is below the battery voltage for almost the entire day. In this case, the boost converter collected approximately 6% more energy than the other two connections. However, the absolute difference in energy collected is trivial.

Although theoretically superior, the boost converter's efficiency advantage of always being able to track the MPP is trivial in practice. However, other advantages may still make this topology a better choice than the buck for a MPPT.

## 3 CONCLUSION

An accurate PV module electrical model is presented and demonstrated in Matlab for a typical 60W solar panel. Given solar insolation and temperature, the model returns a current vector for a given voltage vector.

This model is used to compare the matching efficiencies of three different topologies for connecting a PV module source to constant voltage (battery) load under different conditions of insolation and temperature. The three topologies were a direct PV module - battery connection, a buck converter maximum power point tracker (MPPT), and a boost converter MPPT. The boost converter is shown to have a slight advantage over the buck, particularly at low light levels, since it can always track the maximum power point. The direct connection is always shown to be inferior.

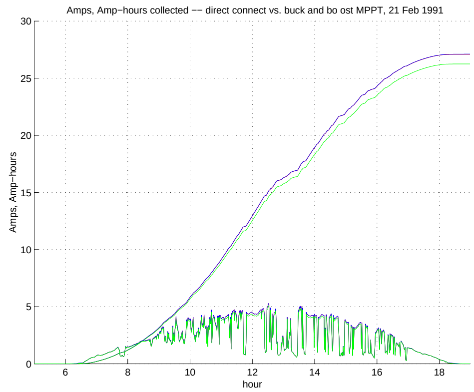


Figure 9: The number of Amps and Amp-hours gathered by the three different matching strategies on Feb 21, at T = $50°$C. The curves for the boost and buck MPPT converters are overlayed (upper trace), with the direct connection curve slightly behind.

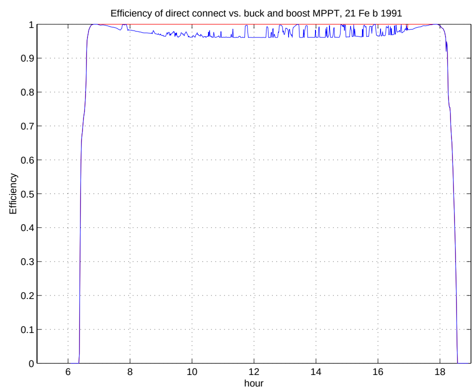


Figure 10: The efficiency of different matching strategies. The boost converter matching is 100%, the buck converter is 100% except for the initial and final low light levels. The direct connect is less than 100% during the course of the day because of mismatch of the MPP and battery voltages.

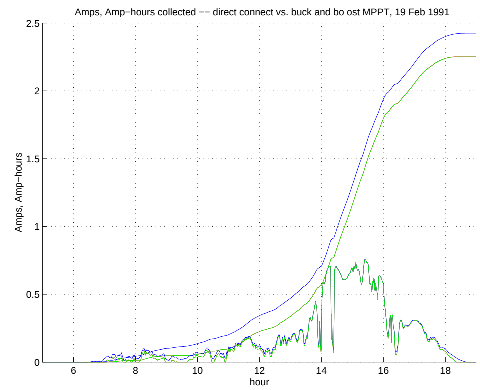


Figure 11: The number of Amps and Amp-hours gathered by the three different matching strategies on Feb 19, at T = $50°$C. This time, the curves for the buck PPPT converter and the direct connection are essentially identical (lower trace), with the boost converter achieving a better result.

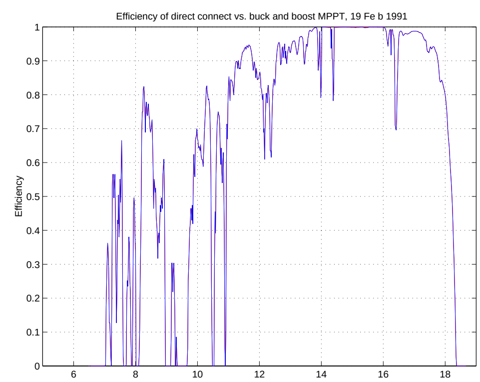


Figure 12: The efficiency of different matching strategies. The direct connection and the buck converter both perform poorly at low light and high temperature conditions. The boost converter always remains 100% matched.

## References


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